ONTARIO

Department of Education

Courses of Study

Grade IX

MATHEMATICS

Issued by Authority of
The Minister of Education

COURSES OF STUDY

FOR

Grade IX (Form I Lower School and Fifth Classes)

IN

Collegiate Institutes, High, Vocational and Continuation Schools and Public and Separate Schools

MATHEMATICS

Objectives and Suggestions :

As many pupils of this grade will discontinue the study of mathematics in a year or two, the teacher should aim at giving them some realization of the place of mathematics in the life of to-day. As a means to this end the following objectives are suggested:

(1) to continue the development of accuracy and facility in the use of numbers;

(2) to develop the capacity for reasoning with abstract concepts and, with this object, to show the advantages of algebraic notation;

(3) to make the pupil conscious of important geometric relations and of the possibility of a logical structure of geometry;

(4) to accustom the pupil to the use of graphical representations as pictorial aids in the interpretation of statistical and other functions;

(5) to cultivate the expression of definite ideas in precise language.

The following suggestions are presented as a guide to teachers:

(a) As the primary result to be aimed at in all teaching of mathematics is the power to draw correct conclusions from given assumptions, the teaching should aim at developing the pupil's skill in analysing problems. Emphasis should be placed on the economy of language and accuracy of thought resulting from algebraic symbolism. Formal algebraic manipulation is a secondary consideration which is justified only by its contribution to the larger aim. Long lists of problems illustrating a single point should be discouraged, but review lists of problems involving a variety of ideas should be used.

(b) As learning is most effective when it is the result of the pupil's own investigation, a large part of the teacher's function is to keep alive the pupil's curiosity. New ideas should be presented not as facts to be proved but rather as problems from which facts are to be discovered.

(c) No pupil is efficient without a trained memory; a judicious use of drill material is therefore necessary. As memory training is most effective when the pupil's interest is maintained, drill problems which have reality to him are to be preferred to purely manipulative work.

(d) As the methods and order of presentation outlined in this course are merely suggestive it is hoped that teachers will adopt methods of correlating the work as a result of their own study and experiments.

OUTLINE OF COURSE

SECTION I. Mensuration: line, square, rectangle, circle, rectangular solid, cylinder.

English units of length; history of these units. Use of ruler in accurate construction and measurement of straight lines.

Use of dividers (or compasses) in transferring lengths. Addition and subtraction of line segments.

Numerical exercises, such as: Express 9″ as a fractional and as a decimal part of a yard and the converse problem. Addition, subtraction, multiplication and division of fractions, such as ½ of $(4\frac{5}{16}'' - 3\frac{7}{8}'')$.

Construction of square and rectangle by ruler, set square, etc. Formulae: $P = 2(l + w)$, $P = 4s$, $A = lw$, $A = s^2$. Problems reviewing tables of square measure. [P = perimeter, l = length, w = width, s = side, A = area.]

Use of compasses in construction of circles. Nomenclature of the circle: centre, radius, diameter, chord, arc, sector. Formulae $c = \pi d$ and $c = 2\pi r$ developed and value of π found experimentally. (Winding strip of paper about cylinder, using ruler and two set-squares as form of calipers, etc.) [c = circumference, d = diameter.]

Use of ruler and set-square to find centre of a circle and in construction of a circle through three given points.

Area of circle and of curved surface of cylinder. Formulae $A = \pi r^2$, $A = 2\pi rh$ developed experimentally. [h = height.]

Volume of rectangular solid and of cylinder. Formulae: $V = lwh$, $V = s^2h$, $V = s^3$, $V = \pi r^2 h$. [V = volume.]

Numerical exercises, such as: If the area of a square is 144 sq. in. find its side and perimeter. Find the difference in perimeter between a square and a rectangle of equal area whose sides are 9′ by 4′. If a cylinder whose diameter is 8″ has the same cubical contents as a rectangular solid $6'' \times 6'' \times 10''$, calculate the height of the cylinder.

Problems without numbers, such as: If the side of a square is tripled, what is the effect on the area? If the edge of a cube is halved, what happens to the volume? What effect on the circumference and on the area of a circle is produced by doubling the radius?

Metric units of length—linear, square and cubic measures (restricted to common units). Units of weight and liquid measure. History of units. Formula $i = 2.54c$ developed by exercises in measurement. [i = inch, c = centimetre.]

Conversion from English to metric units and vice versa.

Problems using both English and metric units, using integers, fractions and decimals, and stressing the advantage of decimal notation in computation.

Simple problems in square and cube root (by method of factoring only).

Indirect use of a formula (solving simple equations but without introducing formal methods or terminology), such as: Given the area and the width, find the length.

Supplementary topics (to be taught if time permits):

History and meaning of other units of length: cubit, span, fathom, nautical mile, etc.

SECTION 2. Decimals and Percentage.

Arabic system of numeration. The decimal point. Comparison of 7.75 and .0075, etc.

Addition, subtraction, multiplication, division and simple roots of decimal fractions.

Short methods of calculation reviewed and extended. Stress need for accuracy. Methods of checking, including checking by estimation.

Practice in expressing vulgar fractions and mixed numbers in decimal notation, and converse problems.

Significance of digits in numbers obtained by measurement and in numbers obtained in calculation from measurements. (Significant figures.)

Problems: decimal notation employed in problems based on measurement and cost of electricity and measurement of density (without formal terminology), expansion per unit length, etc.

Changing decimal and vulgar fractions to percentages, and converse problems.

Problems involving simple applications of percentage. Use of formulae.

Supplementary topics (to be taught if time permits):

History of Arabic system of numeration.

Roman and other systems; methods of performing fundamental operations in these systems.

Expressing fractions with denominators 3, 6, 7, 9 as recurring decimals. What fractions produce terminating decimals? recurring decimals? Maximum number of digits in the period of a recurring decimal.

Expressing a recurring decimal as a fraction.

Composite notations: $7\frac{1}{7}\%$, .08 $\frac{1}{3}$, etc.

SECTION 3. Algebraic Notation, Simple Equations.

Constants: integers and fractions (3 types).

General numbers: use of x, y, z, etc., to represent abstract numbers.

Nomenclature of algebra introduced: coefficient, index, term, expression.

Use of operation symbols and brackets.

Meaning of x^2, xy, m^3, y^2z, etc., made clear by numerical and geometric illustrations.

Correlation of arithmetical and algebraic notations and operations to be stressed.

Equations of first degree in one unknown; practice in constructing and solving such equations by means of axioms; integral, fractional and decimal coefficients to be used; positive roots only; verification of solutions.

Formulae reviewed and extended; calculations based on formulae; construction of tables from formulae.

SECTION 4. Graphs and Simple Geometry.

Graphs:

Use of squared paper; scale drawings based on rectangle and square; numerical problems.

Simple graphs: bar graph, broken-line graph, rectangular distribution graph.

Graphs of simple formulae, such as:

$c = 5n$ (cost of a number of articles at 5c. each).

$c = 25n + 15$ (similar to previous plus a constant, say 15c. for car fare).

$i = 2.54c$. (changing a measurement in centimetres to inches).

$v = kr^2$ (volume of cylinder of any radius).

$1w = 24$ (area of rectangle is constant).

Contrast graphs of these formulae with examples of broken-line graphs, stressing the "smoothness" of the graphs derived from equations.

Geometry: angle, vertex and arms of an angle.

Classification: right, acute (sharp), straight, obtuse (blunt).

Construction and measurement of angles by protractor; unit of measurement.

The degree.

Sum of the angles in any triangle derived by measurement and demonstrated experimentally.

Circular distribution graphs.

Relation of arc and sector to angle at centre of circle.

Exercises reviewing the mensuration of the circle.

SECTION 5. Directed Numbers.

The use of plus and minus as signs of direction or quality made clear by applications to loss and gain, temperatures above and below zero, etc.

Reading of graphs showing monthly balances, temperature records, a ship's course in N. and S. latitudes, etc.

Graphs of positive, zero and negative numbers along a straight line; calculation of distances between two points on the same side or on opposite sides of zero (each example to be illustrated by diagram on graph sheet); integral and decimal notations to be used.

Stress the use of *positive* and *negative* to signify opposite concepts; examples based on motion forward and backward (rugby game), loss and gain, etc., requiring the addition of numbers with like and unlike signs.

SECTION 6. Addition and Subtraction.

Addition and subtraction of expressions not exceeding three terms.

Insertion and removal of brackets in problems involving addition and subtraction.

Practice in simplifying and solving simple equations having positive and negative roots.

Checking of algebraic additions and subtractions by substitution of numerical values.

SECTION 7. Multiplication and Division.

Multiplication and division of directed numbers.

Rule of signs developed.

Review of meaning of positive integral index; formulation of index laws for multiplication and division.

Multiplication (with geometric illustrations) of binomials by monomials and of binomials by binomials, as:

$$a(x + y) = ax + ay;\ (a + b)(x + y) = ax + ay + bx + by;$$

$$(a + b)^2 = a^2 + 2ab + b^2.$$

Division by monomials and binomials.

Application of index laws to powers of integers.

Checking of products and quotients by substitution of positive and negative values.

Analysis of steps employed in multiplying 47×35 and 4.7×3.5 from the product of $(4x + 7)(3x + 5)$; short method of performing arithmetical multiplications of this type.

SECTION 8. Simple Factoring and Application to Operations with Fractions.

Factoring of simplest type forms, such as: common factor, difference of squares of binomial type, trinomials with coefficient of first term unity, including complete squares.

Arithmetical and geometrical applications of factoring, such as: simplifying 93×87, $69^2 - 31^2$, $\sqrt{37^2 - 12^2}$,etc., calculating area of a path or annulus.

Least common multiple.

Reduction, addition, subtraction, multiplication and division of simple fractions; correlation with similar operations using arithmetical fractions.

SECTION 9. Practical Applications of Geometry.

Construction of triangle by ruler and compasses given lengths of three sides.

Classification of triangles by sides.

Construction of triangle by ruler and protractor given two sides and contained angle, or two angles and one side.

Classification of triangles by angles.

Similar triangles; equality of ratios of corresponding sides deduced from measurements; application to calculation of heights and distances.

Pythagorean relationship in right-angled triangle deduced from cutting sum of two squares and tested by measurement.

Construction of the 30°, 60°, 90° triangle and of the 45°, 45°, 90° triangle.

Problems based on Pythagorean theorem, such as: calculation of length of rafter, distance *as crow flies*, etc.

Formal arithmetical square root taught from the identity $x^2 + 2xy + y^2 = (x + y)^2$ (illustrated geometrically); problems limited to exact squares not exceeding six digits.

Use of square root tables; estimation of approximate square roots and checking by multiplication. (The short method of calculation 34.6^2 from 34^2 should be reviewed; also degree of accuracy of the product—significant digits.)

Mensuration of the triangle, parallelogram and trapezoid with general formulae.

Supplementary topics (to be taught if time permits):

Geometrical designs based on straight line and circle.

Construction and use of simple instruments for indirect measurement.

SECTION 10. Equations.

Equations of the first degree in one unknown.

Solution by axioms reviewed.

Transposition rule introduced.

Literal coefficients used in simplest types.

Practice in translating statements into equations and equations into statements.

Problems solvable by first degree equations.

SECTION 11. Geometry and Mensuration.

Accurate constructions with ruler and compasses. Emphasis on ability to describe a construction in good geometrical language.

Bisection of straight lines.

Bisection of angles.

Construction of an angle equal to a given angle.

Drawing of perpendiculars and of parallel straight lines.

Simple inferences based on axioms.

REFERENCE BOOKS AND ALLOTMENT OF TIME

The following estimate of time required for the various sections of the course and references to recommended text-books are for the guidance of the teachers:

Section	Estimated Time Periods	Weeks	References
1	12	3	No text covers this section. The teacher should prepare his own. Help may be had from parts of Mathematics for Technical Schools (The Copp, Clark Co., Ltd.), Chap. IV and pp. 205-6, 208, and from Geometry for High Schools (The Macmillan Co., Ltd.), pp. 5-11.
2	8	2	Mathematics for Technical Schools, Chap. III.
3	16	4	Ontario High School Algebra (The Macmillan Co., Ltd.), Chaps. I and II.
4	5 4 3	3	Mathematics for Technical Schools, Chap. IX, pp. 119-135. Geometry for High Schools, pp. 12-24. Mathematics for Technical Schools, p. 132 and pp. 97-101.
5	4	1	Ontario High School Algebra, Chap. III.
6	8	2	Ontario High School Algebra, Chap. IV.
7	12	3	Ontario High School Algebra, Chap. V.
8	20	5	Ontario High School Algebra, Chaps. VIII and IX.
9	3 9 4	4	Geometry for High Schools, pp. 31-34. Modern Junior Mathematics (D. C. Heath & Co.), Book II, Section 14, pp. 246-258. Mathematics for Technical Schools, pp. 62-66. Mathematics for Technical Schools, pp. 87-91.
10	14	3½	Ontario High School Algebra, Chap. VI.
11	10	2½	Geometry for High Schools, p. 21, pp. 26-38.
Total	132	33	